A quadratic equation is a polynomial of degree 2 and can be written in standard form , where , , and are real numbers and . The solutions to a quadratic equation of the form are called roots or zeros. These are represented on a graph as the -intercepts.

One way to solve quadratic equations is by factoring. But to do that, we must use the Zero Product Property.

If a quadratic polynomial can be factored, the Zero Product Property may be used:

# Zero Product Property

If , then either x = 0 or y = 0.

For example, if *,* then *or .* Solving these two linear equations would give us *or*  as the solutions (or roots or zeros or -intercepts).

Examples: Use the Zero Product Property to solve the following factored equations.

1)

2)

However, most of the time, quadratic equations are not already factored or set equal to 0.

# Solving Quadratic Equations by Factoring

Examples: For each of the following, solve the quadratic equations by factoring.

In certain situations (i.e. when a quadratic equation is in a form which has only one ), we can solve the quadratic equation by isolating the squared term and taking the square root of both sides. This method uses the square root property.

# The Square Root Property

With the term isolated, the square root property states that:

Where is a nonzero real number.

**Solving Quadratics by using the Square Root Property**

Examples: For each of the following, solve the quadratic by using the square root property.

1)

2)

3)

4)

Sometimes a quadratic cannot be factored. We can make it “factorable” using the process of completing the square. Because we are solving, it’s easier to make the equation equal to 0 before beginning. We should also always check to see if a problem is easily factorable first, to make sure completing the square is truly necessary.

# Solving Quadratics by Completing the Square

Before starting, make sure the coefficient of the squared term is 1.

We will use the example .

| Step | Example |
| --- | --- |
| With , first add or subtract the constant term to the right side of the equal sign. |  |
| Find one half of , the coefficient of . | So |
| Square the result of step 2. |  |
| Add the result to both sides of the equation. |  |
| Factor the left, simplify the right. |  |
| Take the square root of both sides and solve the equation. |  |

Examples: Solve each of the following by completing the square.

Another method of solving a quadratic equation is to use the quadratic formula. While factoring and the square root method aren’t always applicable, the quadratic formula can be used to solve ANY quadratic equation.

# The Quadratic Formula

Any quadratic equation written in standard form, , can be solved using the quadratic formula:

Where , , and are real numbers and .

**Solving Quadratics Using the Quadratic Formula**

Examples: For each of the following, solve the quadratic equation by using the quadratic formula. Give your answer in exact values.

1)

2)

3)